

Examining the Effect of Teaching with Ethnomathematics on Students' Problem-Solving Skills on Transformation Geometry*

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ABSTRACT	ARTICLE INFO
This research aims to examine the impact of teaching with ethnomathematics on students' problem-solving skills in transformation geometry and their skills in problem-solving steps. The study was carried out with an experimental design, one of the quantitative research methods. The sample was determined as 42 eighth-grade students in two classes using the purposive sampling method. One of these classes was randomly assigned as the experimental group and the other as the control group. In the research, the "Problem-Solving Test (PST)" was used as a data-gathering tool, and the "Problem-Solving Skill Scoring Key" put forward by Baki (2006) was used in the analysis of the data. According to this key, student responses were graded and the results were noted for the data analysis phase. The results obtained were analyzed with the SPSS20 program. It was seen that teaching using ethnomathematics elements improved students' skills in the steps of understanding the problem, preparing a plan, and plan implementing at the end of the research. The instruction did not have a significant effect on student skills at the evaluation stage. At the same time, teaching using ethnomathematical elements increased students' problem-solving skills and solution skills in daily life problems and traditional problems. In this context, it is recommended to integrate cultural elements into the secondary school mathematics curriculum, and in this way, it is thought	Article History: Received: 01.11.2023 Received in revised form: 24.11.2023 Accepted: 09.12.2023 Available online: 10.12.2023 Article Type: Research Article Keywords: Ethnomathematics, Problem- solving skills, Problem-solving steps
that it can contribute to the development of students' problem-solving skills.	© 2023 JIETP All rights reserved

1. Introduction

Problem-solving skills are one of the most important skills that are given importance to the development of individuals since childhood. It is among the aims of the current secondary school mathematics curriculum that problem-solving skills, which begin to develop at an early age through interaction with the family and the environment, reach a level that will improve the quality of life through mathematics lessons (Ministry of National Education [MoNE], 2018). On the other hand, the first of the five mathematics field skills included in the K12 Turkey Integrative Model, which reveals the 21st-century skills that are aimed to be acquired by students, is expressed as 'Mathematical Problem Solving Skills'.

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Developing problem-solving skills, which is one of the basic skills that the mathematics program aims to provide students with, directly affects mathematics success (Damopolli et al., 2017; Özsoy, 2005). Improving students' mathematical knowledge and skills will both make it easier for education to reach the national vision and contribute to Turkey's prestige in education in the international arena. Two evaluation systems in which mathematics achievement is measured at the international level and in which our country participates are PISA and TIMSS. It is stated that these international evaluation systems aim to measure students' abilities to use the knowledge they learn in mathematics lessons in daily life situations (MoNE, 2020). In this context, it is seen that the importance of associating mathematics with daily life rather than the perception that it is a set of abstract concepts is emphasized by both the curriculum and international evaluation systems. However, in PISA 2018, it is seen that Turkey ranks 42nd among 79 countries in the field of mathematical literacy (MoNE, 2018). It was revealed that Turkey did not show sufficient success in TIMSS at the secondary school level (MoNE, 2020). In this context, it can be said that it is important to develop students' problem-solving skills by associating mathematical knowledge with daily life situations.

Şahin (2004), who mentions the necessity of giving students daily life situations appropriate to their level to develop problem-solving skills, states that students should then review possible solutions and results to the problem. In this regard, the problem-solving process is expressed as a combination of stages in which a person selects and implements the appropriate solution among more than one solution (Phillips et al., 1984). Similarly, Polya (1957), who mentions the four steps of the problem-solving process as understanding the problem, preparing a plan, implementing the plan, and evaluating it, expresses problem-solving as a process in which these steps are followed. In this context, problem-solving competence depends on the successful execution of all these steps.

On the other hand, considering that mathematics has developed in the past centuries with the contribution of many different societies to some common activities (Bishop, 1988), it can be said that mathematics lessons should not be considered separately from societies and their lives and cultures. Especially in recent years, it has been seen that the importance of using various teaching approaches such as realistic mathematics education, out-of-school learning, problem-based teaching, and daily life problem situations in mathematics teaching has been emphasized (Bingölbali et al., 2016). It is also seen that some of these activities are included in the cultural framework and the potential effects of teaching with cultural elements (such as rugs, tiles, mosques, textiles, and weavings) on problem-solving skills are being investigated (Simamora and Saragih, 2019). In this regard, it can be said that it is important to examine the relationships of cultural elements, which form an important part of social life, with mathematical concepts and activities.

The ethnomathematics approach, which attaches importance to associating culture with mathematics, refers to mathematical thoughts and activities in the socio-cultural environment (D'Ambrosio, 1985b). The 'Ethnomathematics Program', which was put forward in the 1980s, aimed to examine students' use of mathematical ideas in daily life problem situations they encounter. (Rosa, 2000). In this context, while it can be seen that mathematics cannot be considered separately from daily life, and daily life from cultural thoughts and activities, it can also be stated that these phenomena are closely related to problem situations.

This study aims to examine the effect of teaching using ethnomathematics activities on students' problem-solving skills in transformation geometry. For this purpose, it aims to investigate the impact of teaching with ethnomathematics on students' skills in problem-solving steps, problem-solving skills, and solution skills in daily life problems and traditional problems. By the aim of the research, answers to the following problems are sought.

1. Does teaching with ethnomathematics have a meaningful effect on students' skills in problem-solving steps in transformation geometry?

2. Does teaching with ethnomathematics have a meaningful effect on students' problemsolving skills in transformation geometry? 3. Does teaching ethnomathematics have a meaningful impact on students' problemsolving skills for everyday problems and traditional problems in transformation geometry?

It is thought that the study is important both because it examines in-depth students' ethnomathematics practices and their development in problem-solving skills, and because it contributes to the limited study in this field in the national literature. At the same time, revealing whether a different teaching approach that can be used within the scope of the national curriculum affects the development of problem-solving ability, which is one of the basic skills for mathematics lessons, gives the study the potential to bring important suggestions for teachers.

2. Theoretical Framework

This section includes the theoretical framework regarding problem-solving and ethnomathematics.

2.1. Problem-Solving

Problem-solving refers to the cognitive process in which a person works towards a specific goal when he does not know what to do in the face of a challenging situation (Schoenfeld, 2013). It is possible to define problem-solving skills as the ability to achieve the appropriate result by carrying out this process successfully. Mayer (1999) defines problem-solving as the cognitive process carried out to reach the desired situation by using given situations when there is no clear path. He states that this process has four components. These; It is expressed as cognitive, process, directional, and personal. Although different definitions have been put forward in many studies in the literature, it is possible to say that problem-solving is a process with the following characteristics.

- Problem-solving refers to the cognitive process passed through to reach a result rather than reaching a result.

- Problem-solving involves actions taken for a specific purpose.

- Problem-solving involves efforts to resolve confusion in a person's mind when the solution method is not clear.

On the other hand, Polya (1957) mentioned four steps of the problem-solving process. For the step of understanding the problem, Polya (2004) emphasizes the importance of the teacher asking the students to express the problem in their own words for the problem to be understood by the students. Schoenfeld (1999), who likens the process of determining a suitable way to solve the problem after understanding the problem to choose the key that will open the door among many keys, states that in this process, the student must choose the appropriate problem-solving strategy. Making a plan phase includes the implementation of the determined strategy. At this stage, when it is thought that the strategy is not suitable, one can go back and think about the chosen strategy (Kayapınar, 2015). In the looking back phase, the main purpose is to check the accuracy of the solution (Ersoy, 2016).

As a result, when the problem-solving process is examined, it can be said that a successful problemsolver should carefully follow every stage of this process. We can say that it is important to examine student skills in detail at all stages of the process of evaluating problem-solving skills.

2.2. Ethnomathematics

The relationship between culture and mathematics is a reality that has existed since the emergence of mathematics. It is known that arithmetic and geometry emerged from daily life situations such as knowing the number of societies' goods and the area of their fields. The first person to consider this relationship as a research topic was Oswald Spingler (Barton, 1999). The first researcher to use the concept of ethnomathematics in its current sense was Ubiratan D'Ambrosio (Achor et al., 2009). Many theoretical and experimental studies have been conducted in this field of research, which gained momentum with the establishment of the 'International Ethnomathematics Group' in the mid-1980s, since its emergence until today.

Gerdes (1994) defines ethnomathematics as a field of research in the cultural anthropology of mathematics and mathematics education. Cultural anthropology aims to understand what exists in a society's culture. For this reason, it can be said that ethnomathematics deals with the relationship of mathematics with cultural anthropology because ethnomathematics takes over the subjects examined within the scope of cultural anthropology by associating them with mathematics. For example, Turkish classical music, rug weaving, Çarpana weavings, blinds, and mosques, which have an important place in our culture, can be discussed within the scope of cultural anthropology. From this perspective, ethnomathematics handles the cultural dimension of mathematics, where cultural anthropology is included as a branch of science. Rosa (2000) defines ethnomathematics as a field consisting of the intersection of mathematics, cultural anthropology, and cultural history, and states that the use of cultural history in mathematics education will help students understand that mathematics is a part of culture. Incorporating ethnomathematical elements into mathematics teaching will allow students to learn in environments they are familiar with from their daily lives, while also helping them feel more accepted in the learning environment. In this context, the benefits of integrating ethnomathematics practices into mathematics teaching have been demonstrated through various studies (Setiyadi et al., 2018; Massarwe et al., 2010; Achor et al., 2009).

On the other hand, the educational dimension of ethnomathematics and the studies carried out in this field are also important. François (2010) describes teaching with ethnomathematics as a process with two aspects. The first of these is that teaching practices cover the curriculum content, and the second is that the prepared content is presented to the student using appropriate ethnomathematics activities. The preparation and teaching processes of the activities in the research were carried out based on this theoretical framework.

When we look at both national and international literature, we come across studies focusing on teaching with ethnomathematics. For example, in the case study by Mania and Alam (2021), in which they aimed to determine the perspectives of teachers in Indonesia regarding the ethnomathematics approach and practices, it was seen that all teachers' attitudes towards ethnomathematics were positive. Teachers stated that ethnomathematics should be used in mathematics teaching. For example, in the study conducted by Nur et al. (2020) with high school students, studies were carried out at the beginning of education to help students understand the relationship between culture and mathematics. Then, the students were asked to produce and solve problems using the cultural elements in their environment. After being allowed to present the examples they found in the class, they were allowed to interpret the work done. In another study, Setiyadi et al. (2018) used games in their culture and associated them with mathematics in their study with 4th-grade students. Researchers who preferred the pretest-posttest design for evaluation concluded that students' problem-solving skills improved with cultural games. In their study with secondary school students, Hartinah et al. (2019) aimed to examine the effect of using the ethnomathematics learning model on students' mathematical communication skills. Three communication activities were prepared based on the curriculum content. During the study process, it was observed that the students did not understand the first activity very well, their understanding improved in the second activity and they started asking questions, and they had a productive learning process in the third activity. As a result, it was found that teaching with ethnomathematics affected mathematical communication skills.

It has been observed that there are studies in the literature that particularly relate ethnomathematics and problem-solving skills. In these studies, it was examined how teaching using ethnomathematical elements affected students' problem-solving skills. As a result, it was concluded that the studies improved the problem-solving skills of the students (Setiyadi et al., 2018; Widada at al., 2019; Nur et al., 2020). However, no national or international study has been found in the literature examining the effect of teaching with ethnomathematics on students' skills in problem-solving steps. In addition, it is seen that ethnomathematical studies in the national literature are newer than the international field. We think that this study is original because it reveals the effect of teaching with ethnomathematics on problemsolving skills through detailed examination.

3. Methodology

3.1. Research Design

This study was carried out with an experimental design, one of the quantitative research methods. Experimental research is research in which comparable procedures are applied and the most precise results can be obtained among scientific methods (Büyüköztürk et al., 2008). In this direction, the study was conducted in 8th-grade transformational geometry in a pretest-posttest control group design.

3.2. Sample of the Study

The participants of the study consist of 42 students studying in two 8th grades at a public secondary school. 21 students are in the experimental group and 21 are in the control group. The participants were determined by typical case sampling from purposive sampling. Typical case sampling refers to the selection of a group of ordinary situations found in the universe for research (Başaran, 2019). Accordingly, it was determined that the participants included students with low, medium, and high achievement levels in line with the previous exam scores of the relevant classes, and the classes were randomly divided into an experimental and a control group.

3.3. Data Collection Tool and Experimental Process

First of all the necessary ethics committee permission was obtained from the publication and ethics committee of the Sakarya University for the study.

The "Problem Solving Test (PST)" developed by the researchers was used as a data gathering. For the development process of the problem-solving test, firstly, the problems prepared in national and international research on transformation geometry were examined. Then, a question pool was created by bringing together the questions obtained from this research and those selected from the transformation geometry questions in the Ministry of Education research textbooks. A 10-question test was prepared by arranging these questions so that the students would study more actively. In the test, to measure student skills in Polya's problem-solving steps, 4 sub-questions were added to each problem to represent the steps of understanding the problem, making a plan, implementing the plan, and looking back.

For the first version of the test, which had 10 questions, expert opinions were received from 3 mathematics education experts and one primary school mathematics teacher. The test is in line with expert opinions; it was developed in the context of design, number of problems, use of visuals, expression of problems and sub-questions, and inclusion of daily life problems. A test consisting of 5 questions was prepared in line with the opinion that the application process was a 1-course period and that it was more appropriate for the test to consist of 5 problems. Although the sub-questions of the test were determined by using similar studies encountered in the literature review, they were rearranged in line with expert opinions to be suitable for the understanding level of 8th-grade students. In this context, the sub-questions in the problems in the test are "Rephrase the question in your own words", "Explain how you will follow to solve the question", "Please solve the problem. You can show your solution by drawing it in the coordinate system above" and "Do you think you solved the question correctly? If your answer is 'Yes', explain why. If your answer is 'No', explain why" (Appendix 1). On the other hand, importance was given to preparing these questions to include the steps of Bloom's Taxonomy. While all 5 problems cover the first five dimensions of Bloom's Taxonomy, only the 2nd problem is associated with the creativity step.

The teaching of the experimental and control groups was carried out in 10 hours, taking into account the teaching program on transformation geometry. A total of 8 ethnomathematical activities, including 6 activities developed by the researchers and 2 activities obtained from previous research, were used in the experimental group, while teaching was continued with the traditional method in the control group. Information about the ethnomathematics activities used in the experimental process is included in Table 1.

Activity	Activity Name	Developer
1	Wonderful View of Egirdir Lake	Researchers
2	Let's Explore Conversions	
	Canvas	Researchers
	Sakarya Couch	
3	Çarpana Weaves	Yazıcı (2021)
4	Examine Aspendos Antique Theater	Researchers
5	A Gift Rug for Osman Bey	Researchers
6	Let's Make Kilis Quilt	Ergene (2022)
7	Design Your Own School's Garden	Researchers
8	Free Cultural Motif Review	Researchers

Table 1. Ethnomathematics Activities Used in the Experimental Group

In most of these activities, students were asked to find reflections and translations of the cultural element given in the activity. In the activity taken from Ergene (2022), the questions were written using the relevant source. In the activity taken from Yazıcı (2021), the questions were created using the relevant source. In this context, students were also asked to make designs in some activities (A Gift Rug for Osman Bey, Let's Make Kilis Quilt, Design Your Own School's Garden).

Before the application, PST was applied to measure the groups' preliminary knowledge, and after the instruction, the same test was applied as a posttest. Information about the teaching carried out in both groups is included in Table 2.

Table 2. Experimental Process Performe	d in Experimental and Control Groups
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Group	Pretest	Teaching Activities	Posttest
Experimental Group	PST	Ethnomathematics Activities	PST
Control Group	PST	Traditional Expressions	PST

In the first lesson of the ten-hour teaching process, studies were carried out to understand the transformations by showing translation and reflection examples from daily life (car moving forward, flag rising on the pole, visuals for reflection) through a presentation prepared for experimental group students. In the second lesson, these examples were selected from cultural elements (tile motifs, rug patterns), and the relationship between culture and transformations was explained. In this narrative, the symmetry of the tile motifs was used and the translations and reflections of the rug patterns were emphasized. In this way, the relationship between culture and mathematics was tried to be structured in the student's mind.

Activities were used in addition to the curriculum content, with one activity in each subsequent lesson. In these lessons, firstly, activity sheets were distributed to the students, and students were asked about the cultural element in the activity: 'Have you seen this element before?' 'Where did you see?' Questions such as these were asked and information was given about these items. Afterward, students were given time to answer the questions given in the activity. Then, a discussion environment was created for volunteering students to explain their answers. In this process, students expressed their opinions and reflected on why their answers were correct or incorrect. At the end of the lesson, the teacher ensured that generalizations within the scope of the subject were reached through the activity. Activity sheets were collected from students at the end of each lesson.

In the control group, the traditional teaching method, which has been applied for many years and can also be expressed in the form of direct instruction, was used. What is meant by traditional here is teaching in which the teacher is at the center and no different teaching materials are used.

3.4. Analysis of Data

The "Problem-Solving Skill Scoring Scale (Appendix 2)" developed by Baki (2006) was used to score student answers to the PST. In the evaluation made with this scoring scale, student answers were graded

with 0, 1, 2, and 3 points according to the proficiency revealed for the sub-questions of each problem. The scoring for the problem-solving steps within the framework of Baki's (2006) scoring rubric is as follows:

- 3 points: If the student was able to fully demonstrate the skill in the problem-solving step, he received 3 points from that step. For example, at the stage of understanding the problem, the students must explain the problem correctly with new words.

- 2 points: For students to get 2 points from the problem-solving steps, they must do some of what is asked in this step. In other words, an incomplete answer that was not incorrect was evaluated with 2 points. For example, choosing a part of the strategy to be used in the solution is a situation in which this scoring will be received.

- 1 point: If 1 point is received from the problem-solving step, an inappropriate answer must have been given. For example, making an inappropriate solution, not being able to verify the results, and not understanding the problem causes the student to receive 1 point.

- 0 point: Situations where a score of 0 is obtained from the problem-solving step indicate that no action is taken. For example, not making an effort to understand the problem, not making any solution, and not knowing how to evaluate the solution are examples of these situations. For example, in the evaluation step, 'yes' and 'no' answers were evaluated with 0 points because they did not indicate a logical effort and expression regarding the verification of the solution.

The SPSS program (Statistical Package for the Social Sciences) 20 was used to analyze the data obtained.

3.5. Validity and Reliability

In the preparation of the problems in PST, firstly, national and international sources were examined. For the validity and reliability of the test, expert opinions were taken from 3 mathematics education experts and 1 secondary school mathematics teacher. Expressions used in similar studies were used for the sub-questions of the problems. While preparing the questions, care was taken to ensure content validity for all of the outcomes, and a table of specifications was prepared. In this way, the questions in the test were arranged and its reliability was ensured. In addition, in line with the expert opinions received, expert opinions were taken for content validity and face validity. For construct validity, the test includes questions that aim to reveal student skills in the problem-solving steps that are intended to be measured. Expert opinion was taken to determine these sub-questions.

In quantitative research, validity is divided into two: internal validity and external validity. Internal validity refers to explaining that the change in the dependent variable is caused by the independent variable, and external validity refers to the generalizability of the change in the sample to the universe (Büyüköztürk et al., 2008). To ensure the internal validity of the study, information was obtained from the mathematics teacher of the classes to compare the groups' prior knowledge of the subject, and it was checked if there was a significant difference between the classes' prior knowledge on this topic by applying PST. In addition, to prevent the learning processes from being negatively affected, the applications were carried out by the classroom mathematics teacher as usual. For the external validity of the study, the experimental and control groups were randomly assigned, and care was taken to include students with low, medium, and high achievement levels in both classes to increase the generalizability of the results. Additionally, both classes were not informed that they were in an experimental situation.

4. Findings

This section includes the findings obtained in line with the problems of the research.

4.1. First Problem:

"Does teaching based on ethnomathematics have a significant effect on students' skills in problem-solving steps?"

4.1.1. Findings regarding the understanding step of the problem in PST

Concerning the question "Rephrase the question in your own words," the first of the sub-questions on the tasks in the PST, the mean values obtained for both groups throughout the test from the problem comprehension step do not show a normal distribution. The Mann-Whitney U Test results applied in this direction are shown in Table 3.

una contra	n Groups						
Test	Group	Ν	Mean Rank	Sum of Ranks	Mann-Whitney U	Sig.	
Pretest	Experimental Group	21	24,52	515	157	,108	
	Control Group	21	18,48	388			
Posttest	Experimental Group	21	28,05	589	83	,000**	
	Control Group	21	14,95	314			

Table 3. Mann-Whitney U Test Results about the Problem Understanding Level of the Experimental and Control Groups

* p<.05, ** p<.01

According to Table 3, while there is no meaningful difference between the results of the experimental group and the control group regarding the problem understanding step in the pretests, it is seen that there is a meaningful distinction between the mean scores in the posttests in favor of the experimental group (p < 0.05). In other words, there is no significant difference between the understanding of the problems related to the problems given on transformation geometry in the experimental and control groups. However, after the instruction, the experimental group's ability to understand problems in transformation geometry increased significantly compared to the control group.

4.1.2. Findings regarding the PST making a plan step.

Since there was a meaningful difference between the results of the experimental group and the control group in the pretest regarding the question "Explain how you will follow to solve the question", which is the second of the sub-questions to the problems in PST, ANCOVA was used for making a plan score averages obtained from all the problems in the test for the groups. The results of the ANCOVA test are included in Table 4.

Courses	Sum of	64	Mean	Б	C: ~	Eta-
Source	Squares	5u	Squares	Г	51g.	Squared
Pretest	,209	1	,209	,025	,876	,001
Group	135,51	1	135,51	16,02	,000**	,291
Error	329,88	39	8,45	-	-	-
Total	5957	42	-	-	-	_

Table 4. ANCOVA Test Results about the Making Plan Step of the Both Groups

* p<.05, ** p<.01

According to Table 4, we can say that there is a meaningful distinction between the results of both groups in the posttest, which is a plan step in favor of the experimental group (p < 0.05). During the plan preparation phase, it was seen that the experimental group chose a solution strategy after teaching with ethnomathematics more successfully than the control group.

4.1.3. Findings regarding the PST plan implementation step.

Regarding the question "Please solve the problem. You can show your solution by drawing it in the coordinate system above.", which is the third of the sub-questions to the problems in the PST, the mean scores obtained from the plan implementation step for both groups do not show a normal distribution. The Mann-Whitney U Test results applied in this direction are shown in Table 5.

Test	Group	Ν	Mean Rank	Sum of Ranks	Mann-Whitney U	Sig.	
Pretest	Experimental Group	21	22,55 473 198		,526		
	Control Group	21	20,45	429			
Posttest	Experimental Group	21	30,67	30,67 644	28	,000**	
	Control Group	21	12,33	259			

Table 5. Results of the Mann-Whitney U Test for the Plan Implementation Step of the Groups

* p<.05, ** p<.01

According to Table 5, while the experimental and the control group's results regarding the plan preparation step in the pretest do not show a meaningful difference, it is seen that there is a meaningful difference in the mean scores in the posttest in favor of the experimental group (p < 0.05). During the implementation phase of the plan, in other words, in solving the problem, it was observed that the solution skills of the experimental group increased significantly compared to the control group after the lessons.

4.1.4. Findings regarding the PST looking back step

Regarding the question "Do you think you solved the question correctly? If your answer is 'Yes', explain why. If your answer is 'No', explain why", which is the fourth of the sub-questions to the problems, the mean scores obtained from the looking back step for the experimental and control groups do not show a normal distribution. The Mann-Whitney U Test results applied in this direction are shown in Table 6.

Test	Group	Ν	Mean Rank	Sum of Ranks	Mann-Whitney U	Sig.
Pretest	Experimental Pretest Group 21		22,50	472,5	199,5	,152
	Control Group	21	20,50	430,4		
Posttest	Experimental Group	21	22,12	464	207,5	,561
	Control Group	21	20,88	438		

Table 6. Results of the Mann-Whitney U Test for the Look-Back Step of the Groups

According to Table 6, it can be seen that there is no meaningful distinction between the results of the groups in terms of the look-back step in both the pretests and the posttests (p>0.05).

It is thought that giving examples of students' answers in the looking back step would be more descriptive for this section. No student response was found who received 3 points from the Looking back step. In this regard, Image 1 shows an example of a student's answer who received 2 points from the locating back step.

d. Soruyu doğru çözdüğünüzü düşünüyor musunuz? Cevabınız 'Evet' ise nedenini açıklayınız. Cevabınız 'Hayır' 2) ise nedenini açıklayınız. Evet Conki Konomlorinibul portode gieli üchen beldum

Image 1. The answer of One Student Who Got 2 Points from the Looking Back Step

In this example, the student stated that the reason why his answer was correct was that he found a hidden triangle to make the calculation and made the calculation using it. Therefore, since it was a correct evaluation, it was evaluated with 2 points. Similarly, the answer of a student who got 1 point from the looking back step is given in Figure 2.





The student tried to evaluate his solution in this answer. Accordingly, he stated that his solution was correct. The student partially confirmed her result by stating that she did the steps mentioned. For this reason, the student was given 1 point for this answer.

The student who got 0 points from the looking back step did not give any answer because he did not know how to verify his result. For this reason, it received 0 points from the looking back step.

4.2. Second Problem:

"Does teaching with ethnomathematics have a significant effect on students' problem-solving skills in transformation geometry?"

The total score obtained from PST was evaluated as the problem-solving skill score. Since the results of the problem-solving pretests of the groups did not show a normal distribution, the Mann-Whitney U test results are given in Table 7.

Test	Group	Ν	Mean Rank	Sum of Ranks	Mann-Whitney U	Sig.
Pretest	Experimental Group	21	24,71	519	153	,088
	Control Group	21	18,29	384		

Table 7. Mann-Whitney U Test Results Regarding Problem Solving Skill Pretest Results of the Groups

In Table 7 we can see that there is no significant difference between the problem-solving pretests' results of the experimental and the control group (U= 153; p>0.05). That is, the problem-solving skills of the experimental and control group students on transformation geometry are not different in the pretest.

Since the results of the problem-solving posttests of the groups show a normal distribution, the t-test results for the independent samples are included in Table 8.

Table 0. 1-10	Table 6. 1-10st Results about the 110bent-501ving 5kin 10st-10st Results of the Groups								
Variable	Posttest	Ν	Х	S	t	Sig.			
Problem- Solving Skill	Experimental Group	21	37,23	5,45	(22	000**			
	Control Group	21	23,42	7,50	6,82	,000**			

Table 8. T-Test Results about the Problem-Solving Skill Post-Test Results of the Groups

* p<.05, ** p<.01

Table 8 shows that there is a meaningful distinction between the problem-solving posttests of both groups in favor of the experimental group (p < 0.05). These findings show that the problem-solving skills of the experimental group students increased significantly compared to the control group at the end of teaching with ethnomathematics.

4.3. Third Problem:

" Does teaching ethnomathematics have a meaningful impact on students' problem-solving skills for every day and traditional problems in transformation geometry?"

Since the scores of the experimental and control groups on PST daily life problems did not show a normal distribution, the results of the Mann-Whitney U test can be found in Table 9.

Test	Group	Ν	Mean Rank	Sum of Ranks	Mann-Whitney U	Sig.	
Pretest	Experimental Group	21	23,10	485	187	,397	
	Control Group	21	19,90	418			
Posttest	Experimental Group	21	31,05	652	20	,000**	
	Control Group	21	11,95	251			

Table 9. Results of the Mann-Whitney U Test on the Everyday Problems of the Groups

* p<.05, ** p<.01

According to Table 9, while there is no meaningful distinction between the results of the groups on the PST problems of daily living, there is a meaningful difference in the posttest mean scores in favor of the experimental group (p < 0.05).

Since there was a meaningful distinction between the results of the experimental group and the control group on the traditional PST problems, the ANCOVA test results are given in Table 10.

Source	Sum of Squares	Sd	Mean Squares	F	Sig.	Eta-Squared
Pretest	365,5	12	,209	2,103	,052	,474
Group	447,84	1	447,84	30,92	,000**	,525
Error	405,54	28	14,48	-	-	-
Total	13664	42	-	-	-	-

Table 10. ANCOVA Test Results of the Groups on Traditional Problems

* p<.05, ** p<.01

According to Table 10, it turns out that there is a significant difference between the results of the experimental group and the control group on the traditional PST problems, in favor of the experimental group (p<0.05).

5. Results

In the study, it was observed that;

- Students' skills in understanding the problem, preparing a plan, and implementing the plan in the problem-solving steps in transformation geometry,
 - Problem-solving skills in transformation geometry,
 - Problem-solving skills in daily life problems and traditional problems

improved at the end of teaching with ethnomathematics. It was previously stated that student responses were analyzed to examine students' skills in PST. We said that the total score obtained from the problemsolving test is also considered a problem-solving skill. In particular, the development of students' problem-solving skills in transformation geometry was seen as follows. For example, in the pretest of the problem understanding step, the student stated that he did not understand the problem or used expressions that showed that he misunderstood. In the posttest, he was able to express the problem correctly in his own words at the problem-understanding stage of the same problem. Similarly, it was observed that students who answered 'I don't know' during the plan preparation step were largely able to solve the problem in the posttest. While many students could not solve the problem in the pretest, most of the students were able to solve the problem correctly in the posttest. Although no significant change was observed in the evaluation step, it can be said that the problem-solving skills of the students increased as the performances of the students in the problem-solving steps in the entire test increased and the statistical analysis revealed this difference.

However, teaching with ethnomathematics did not change students' skills at the evaluation stage in the problem-solving process.

6. Discussion

We can see that teaching using ethnomathematics activities increased students' skills in the steps of understanding the problem, creating a plan, and implementing the plan. According to this result, it is possible to say that the use of cultural elements in teaching makes it easier for students to understand the problem and, accordingly, helps them to show higher success in solving problems. Similarly, in the literature, studies conducted with secondary school students show that mathematics teaching supported by ethnomathematics increases students' problem-understanding, planning, and problem-solving skills (Nursyahidah et al., 2018; Supriadi, 2019).

It was concluded that teaching with ethnomathematics had no effect on students' skills at the evaluation stage in the problem-solving process at the end of the study. In their study, Nursyahidah, Saputro, and Rubowo (2018) revealed that teaching with ethnomathematics did not affect students' ability to evaluate their solutions. It is thought that this result is because students have not done any effective work before to evaluate their problem solutions. In this regard, it is thought that before research student skills in problem-solving steps will be measured, checking that the student has sufficient readiness for what to do in these steps will contribute to the process.

Another result of the research showed that teaching using ethnomathematics applications improved students' problem-solving skills in transformation geometry. In their study, Irawan et al. (2018) concluded that real-life teaching practices associated with ethnomathematics contribute to the development of students' problem-solving skills. Similarly, Widada et al. (2019) observed in their study that students' problem-solving skills improved as a result of outdoor education practices in which ethnomathematical elements were used.

In the study, we can say that students' problem-solving skills on everyday problems and traditional problems increased on the test. It is thought that this situation arises because ethnomathematics activities are linked to daily life, making it easier to associate mathematics with daily life, and supporting understanding of traditional problems.

7. Suggestions

It was seen that teaching with ethnomathematics improved students' problem-solving skills in transformation geometry and increased their skills in the problem-solving process. In this regard, it can be recommended that teachers use the ethnomathematics approach as a supportive method in mathematics teaching. By using cultural elements in mathematics teaching, students' processes of making sense of mathematics can be facilitated. When daily life examples are needed in mathematics teaching, choosing examples that contain cultural elements can be preferred as an effective method to keep students' cultural awareness high.

In future studies, the effects of teaching with ethnomathematics on learning can be demonstrated more broadly by applying it to different subjects and outcomes. Student skills and behaviors can be examined in depth by designing multifaceted teaching processes using different representations and models using ethnomathematical elements. Within the scope of out-of-school learning, cultural structures in the cities where schools are located can be visited and associated with mathematics. In this way, mathematics teaching can be enriched.

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Appendix 1: Problem-Solving Test (PST)

Aynı mahallede oturan ve yakın arkadaş olan Kerim ve Onur hafta sonu lunaparka gitmek için anlaşmışlardır. Hafta sonu geldiğinde buluşma için Kerim (-5,3) noktasından, Onur ise (6,-4) noktasından yola çıkarak birbirlerine doğru yürümeye başlamışlardır.

Kerim 2 metre sağa, 2 metre aşağı yürüdüğünde Onur da 1 metre yukarı, 2 metre sola ilerlemiştir. Bu durumda Kerim ile Onur arasındaki en kısa uzaklık kaç metredir?

- a. Soruyu kendi cümlelerinizle yeniden ifade ediniz.
- b. Soruyu çözmek için nasıl bir yol izleyeceğinizi açıklayınız.
- c. Sorunun çözümünü yapınız. Çözümünüzü yukarıdaki koordinat sisteminde çizerek gösterebilirsiniz.



Yukarıda koordinat düzleminde AB ve CD doğru parçaları verilmiştir. Bu doğru parçalarının orta noktalarının (1,-1) noktasında kesişmesi için AB ve CD doğru parçalarına uygulanacak öteleme dönüşümlerini bulunuz.

- a. Soruyu kendi cümlelerinizle yeniden ifade ediniz.
- b. Soruyu çözmek için nasıl bir yol izleyeceğinizi açıklayınız.
- c. Sorunun çözümünü yapınız. Çözümünüzü yukarıdaki koordinat sisteminde çizerek gösterebilirsiniz.

3. Öğrencilerine dönüşüm geometrisi konusunu anlatan Feyza öğretmen, öğrencilerin daha iyi anlaması için okulun bahçesine koordinat sistemi çizerek öteleme ve yansıma hareketlerini gösteriyordur. Yansıma dönüşümüne örnek vermek isteyen Ali bisikleti ile koordinat sisteminin üzerine gelerek durmuştur. Ön tekerleği O(7,-4) noktasında bulunan ve ön ve arka tekerleği arasında yatayda 5 birim mesafe olan Ali'nin bisikletinin direksiyon yönü doğuya doğrudur.



Buna göre Ali bisikletini y eksenine göre yansıtırsa arka tekerleğinin bulunduğu noktanın koordinatları ne olur? (Tekerleğin bulunduğu nokta, tekerleğin yere temas ettiği noktayı ifade etmektedir)



a. Soruyu kendi cümlelerinizle yeniden ifade ediniz.

b. Soruyu çözmek için nasıl bir yol izleyeceğinizi açıklayınız.

c. Sorunun çözümünü yapınız.

4. Bir ABCD dörtgeninin köşe noktalarının koordinatları A(2,-1), B(4,3), C(2,-5) ve D(0,-3)' tür. Bu dörtgen önce xeksenine sonra da y-eksenine göre yansıtılırsa köşe noktalarının yeni koordinatları ne olur?

a. Soruyu kendi cümlelerinizle yeniden ifade ediniz.

b. Soruyu çözmek için nasıl bir yol izleyeceğinizi açıklayınız.

c. Sorunun çözümünü yapınız.



Yukarıda koordinat sisteminde ABC ve DEF üçgenleri verilmiştir. ABC üçgeninin, DEF üçgeninin bulunduğu yere ve şekle gelmesi için ABC üçgenine uygulanması gereken dönüşümleri belirleyiniz.

- a. Soruyu kendi cümlelerinizle yeniden ifade ediniz.
- **b.** Soruyu çözmek için nasıl bir yol izleyeceğinizi açıklayınız.
- c. Sorunun çözümünü yapınız. Çözümünüzü yukarıdaki koordinat sisteminde çizerek gösterebilirsiniz.
- **d.** Soruyu doğru çözdüğünüzü düşünüyor musunuz? Cevabınız 'Evet' ise nedenini açıklayınız. Cevabınız 'Hayır' ise nedenini açıklayınız.

A. Anlama	Problemi	
		3 Problemin tam olarak anlaşılması
		2 Problemin bir parçasının anlaşılması
		1 Problemin anlaşılamaması
		0 Problemin anlaşılması için herhangi bir çabanın gösterilmemesi
B.	Plan Hazırlama	
(Bir Strate	eii Secme	
(2.1 0.1 0.1	<u> </u>	3 Uygun çözüme ulaştıracak bir stratejinin seçilmesi
		2 Çözüme yardımcı olacak stratejinin sadece bir parçasının
		seçilmesi 1
		Livgun olmavan bir stratejinin secilmesi
		0
		Herhangi bir stratejinin seçilmemesi
С.	Planı Uygulama	
		3 Uygun ve doğru çözüme ulaşılması
		2 Bir kısmı doğru olan bir çözümün yapılması
		1 Uygun ve doğru olmayan bir çözümün yapılması
		0 Herhangi bir çözümün yapılamaması
D.	Değerlendirme	
		3 Problemin ve bu probleme göre oluşturulan yeni problemin çözülmesi
		2 Sonuçların mantıksal olarak doğrulanması
		1 Sonuçların kısmen doğrulanması
		0 Sonucun nasıl doğrulanacağının bilinmemesi

Appendix 2: Problem-Solving Skill Scoring Scale (Baki, 2006)

Appendix 3: An Activity Sample from Ethnomathematics Activities

Activity 5. A Gift Rug for Osman Bey



1299 yılında Osmanlı Devleti'ni kuran Osman Bey, bir sefer hazırlığındadır. Bu sefer için herkes elinden geldiğince Osman Bey ve ordusuna yardım etmektedir. Osman Bey'i oğlu gibi seven Fatma Hanım bu sefer için ilerlemiş yaşına rağmen bir şeyler yapmak istemektedir. Uzun bir süre ne yapabileceğini düşünen Fatma Hanım sonunda Osman Bey'in sefer sırasında çadırında kullanması için bir kilim dokumaya karar verir. Bunun için gençken dokuduğu kilimlerden örnek çıkarmak ister.

Yukarıda Fatma Hanım'ın dokumaya karar verdiği kilimi görmektesiniz. Ancak Fatma Hanım'ın yeşil çizgi ile sınırlandırılmış bölgeyi dokurken kafası karışmıştır. Birkaç yanlış dokumanın ardından nerelerde aynı motifi dokuyacağını, nerede simetrik motifler yapması gerektiğini dokuma öncesinde belirlemesi gerektiğini fark etmiştir.

- a. Fatma Hanım'a yardımcı olmak için motiflerde var olan yansıma ve öteleme dönüşümlerini bulup yansıma eksenlerini çiziniz.
- b. Aşağıda Türk kilimlerinde kullanılan bazı motifleri görmektesiniz. Bu motifleri veya iletmek istediğiniz mesajı barındıran sizin tasarlayacağınız yeni motifleri kullandığınız bir kilim tasarlayınız. Tasarladığınız kilimde kullandığınız öteleme ve yansıma dönüşümlerini açıklayınız.





